

## Math 62      Work Rates

### Objectives

- 1) Work rates

### Examples

Solve.

- 1) A painter can finish painting house in 5 hours. Her assistant takes 7 hours to finish the same job. How long would it take for them to complete the job if they were working together?
- 2) One pump can drain a pool in 9 minutes. When a second pump is also used, the pool only takes 4 minutes to drain. How long would it take the second pump to drain the pool if it were the only pump in use?
- 3) One conveyor belt can move 1000 boxes in 7 minutes. Another can move 1000 boxes in 10 minutes. If another conveyor belt is added and all three are used, the boxes are moved in 3 minutes. How long would it take the third conveyor belt along to do the same job?
- 4) A baker can decorate the day's cookie supply four times as fast as his new assistant. It takes 16 minutes for them to decorate the day's cookie supply if they work together. How long does each one take if working alone?
- 5) Mark and Rachel both work for Smith Landscaping Company. Mark can finish a planting job in 2 hours, while it takes Rachel 4 hours to finish the same job. If Mark and Rachel will work together on the job, and the cost of labor is \$40 per hour, what should the labor estimate be? (Round to the nearest cent, if necessary.)

## Work rates

① If the painter takes 5 hours to do the entire job, the painter can do  $\frac{1}{5}$  of the job in one hour, or .2 job/hour. This is the painter's work rate.

The assistant's work rate is  $\frac{1}{7}$  job per hour.

Method 1: The unknown work rate is based on the unknown time,  $x$  hours to complete the job together. This makes that work rate  $\frac{1}{x}$ , or the fraction they do in one hour.

Equation: add fractions done alone in one hour to get fraction done together in one hour

$$\frac{1}{5} + \frac{1}{7} = \frac{1}{x}$$

$$35x \cdot \frac{1}{5} + 35x \cdot \frac{1}{7} = 35x \cdot \frac{1}{x}$$

$$7x + 5x = 35$$

$$12x = 35$$

$$x = \frac{35}{12} \text{ hrs}$$

$$x = 2.9\overline{16} \text{ hours}$$

multiply by LCD = 35x

cancel  $\frac{35x}{5} = 7x$

$$\frac{35x}{7} = 5x$$

$$\frac{35x}{x} = 35$$

Because working together means working more quickly, the time together should be smaller than either time alone.

Method 2: Multiply each work rate by the time worked together to get work done by each. Add to get one (1) job.

$$\frac{x}{5} + \frac{x}{7} = 1$$

$$35 \cdot \frac{x}{5} + 35 \cdot \frac{x}{7} = 35 \cdot 1$$

$$7x + 5x = 35$$

multiply by LCD = 35

cancel

Continue as in Method 1.

② one pump 9 min  $\Rightarrow \frac{1}{9}$  of job alone

both ("also") 4 min  $\Rightarrow \frac{1}{4}$  of job together

second pump  $x$  min  $\Rightarrow \frac{1}{x}$  of job alone

Method 1:  $\frac{1}{9} + \frac{1}{x} = \frac{1}{4}$       LCD =  $36x$

$$4x + 36 = 9x$$

$$36 = 5x$$

$$7.2 = \frac{36}{5} = x$$

$$\boxed{7.2 \text{ min}}$$

Method 2:  $\frac{4}{9} + \frac{4}{x} = 1$       LCD =  $9x$

$$4x + 36 = 9x \quad \text{cont. as before.}$$

③ one belt 7 min  $\Rightarrow \frac{1}{7}$  of job alone

another belt 10 min  $\Rightarrow \frac{1}{10}$  of job alone

3rd belt, all together 3 min  $\Rightarrow \frac{1}{3}$  of job together

$x$  = 3rd belt time alone  $\Rightarrow \frac{1}{x}$  of job alone

Method 1:  $\frac{1}{7} + \frac{1}{10} + \frac{1}{x} = \frac{1}{3}$       LCD =  $210x$

$$30x + 21x + 210 = 70x$$

$$51x + 210 = 70x$$

$$210 = 19x$$

$$\frac{210}{19} = x$$

$$\boxed{\frac{210}{19} \text{ min}}$$

check  
 $\frac{210}{19} \approx 11.05 \text{ min.}$

Method 2:  $\frac{3}{7} + \frac{3}{10} + \frac{3}{x} = 1$       LCD =  $70x$

$$30x + 21x + 210 = 70x$$

cont as before

④ Baker 4x as fast  $\Rightarrow \frac{1}{4}$  the time!

$x = \text{time} = \text{time for baker} \Rightarrow \frac{1}{x}$   
 $4x = \text{bigger time} = \text{time for assistant} \Rightarrow \frac{1}{4x}$   
 $16 = \text{time together} \Rightarrow \frac{1}{16}$

Method 1

$$\frac{1}{x} + \frac{1}{4x} = \frac{1}{16} \quad \text{LCD} = 16x$$

$$16 + 4 = 4x$$

$$\boxed{20 \text{ min} = x = \text{baker}}$$

$$4x(20) = \boxed{80 \text{ min} = \text{assistant}}$$

Method 2:

$$\frac{16}{x} + \frac{16}{4x} = 1 \quad \text{LCD} = 4x$$

$$64 + 16 = 4x$$

$$80 = 4x$$

$$20 = \frac{80}{4} = x$$

$$\boxed{x = 20 \text{ min Baker}} \\ \boxed{4x = 80 \text{ min assistant}}$$

⑤ Mark - 2 hrs

Rachel - 4 hrs

Together -  $x$  hrs

\$40 per hour (per person!)

Method 1

$$\frac{1}{2} + \frac{1}{4} = \frac{1}{x} \quad \text{LCD} = 4x$$

$$2x + x = 4$$

$$3x = 4$$

$$x = \frac{4}{3} \text{ hrs}$$

Method 2:

$$\frac{x}{2} + \frac{x}{4} = 1 \quad \text{LCD} = 4$$

$$2x + x = 4 \quad \text{cont as before}$$

$$\frac{4}{3} \text{ hrs} \times \$40/\text{person} \times 2 \text{ people}$$

$$= 106 \frac{2}{3} = \boxed{\$106.67}$$